*Journal of Sound and Vibration* (1997) **200**(4), 531–532

# LETTERS TO THE EDITOR

## COMMENTS ON "FREE VIBRATION OF A RECTANGULAR PLATE LOADED BY A NON-UNIFORM IN-PLANE FORCE"

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(*Received* 20 *February* 1996)

The authors have attempted a solution to an important type of elastodynamics problem [1] for which a limited number of studies is available.

Unfortunately, their equation (1) is not valid, in general, when non-uniform normal in-plane forces are applied at the edges $x = 0, a$ (see Figure 1).

In general, the governing differential equation is:

$$D\nabla^4 W - \left(N_x \frac{\partial^2 W}{\partial x^2} + 2N_{xy}\frac{\partial^2 W}{\partial x \partial y} + N_y \frac{\partial^2 W}{\partial y^2}\right) - \omega^2 \rho h W = 0, \tag{1}$$

where $N_x = \sigma_x h$, $N_y = \sigma_y h$ and $N_{xy} = \tau_{xy} h$. The three components of the stress tensor corresponding to the plane stress problem are determined solving the well known equation

$$\nabla^4 U = 0 \tag{2}$$

where $U$ is Airy's stress function,

$$\sigma_x = \frac{\partial^2 U}{\partial y^2}, \qquad \sigma_y = \frac{\partial^2 U}{\partial x^2}$$

and

$$\tau_{xy} = -\frac{\partial^2 U}{\partial x \partial y}.$$

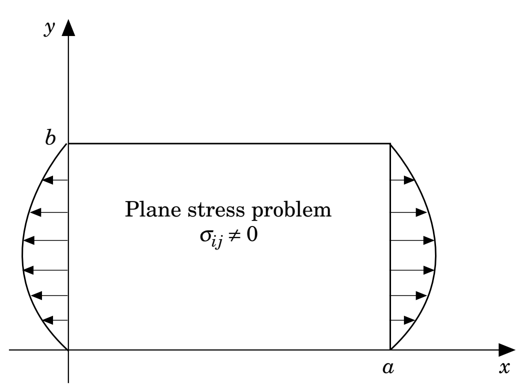


Figure 1. A rectangular plate subject to non-uniform in-plane force executing transverse vibrations.

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Consequently, one must solve first the plane stress problem. Once the stress field is known, one must replace the authors' values in equation (1) and then solve the complete partial differential equation. Obviously, the problem solved by the authors is not the correct one, in general, when the applied forces are not uniform.

Admittedly, the results obtained by the authors are in good agreement with finite element predictions, see Table 1 of [1]. Fortunately, in this case the assumed stress distribution is the correct one according to the mathematical theory of planar elasticity.

REFERENCES

1. S. Kukla and B. Skalmierski 1995 *Journal of Sound and Vibration* **187**, 339–343. Free vibration of a rectangular plate loaded by a non-uniform in-plane force.

## AUTHORS' REPLY

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(*Received* 10 *May* 1996)

The authors wish to thank Professor Laura, Dr Avalos and Dr Larrondo for their comments [1] on the authors' letter [2]. Our letter deals with free vibration problem of an in-plane loaded rectangular plate with two opposite edges simply supported. In the case of linearly varying normal forces which act at the simply supported edges, the approach fully takes into consideration the state of stress in plane of the plate. In other cases, the formulation of the problem is based on an simplified assumption on the stress state. This assumption permits the direct method of solution which is presented in the paper [2].

The solution of the biharmonic equation (2) mentioned in the comments [1] with the boundary conditions assumed in the paper [2] can be obtained by using an approximate method. Timoshenko and Goodier [3] have presented the approximate form of the Airy function for this problem which is obtained by the application of the principle of least work. When the distribution of stresses is known, an approximate method can be applied to obtain the solution of the vibration problem.

The authors appreciate the comments of Professor Laura *et al*. We will address the subject of vibration of plates with a more complicated state of stress, in due time. At this time, we refer to the results of the letter [2].

REFERENCES

1. P. A. A. Laura, D. R. Avalos and H. A. Larrondo 1996 *Journal of Sound and Vibration* **200**, 531–532. Comments on "Free vibration of a rectangular plate loaded by a non-uniform in-plane force".
2. S. Kukla and B. Skalmierski 1995 *Journal of Sound and Vibration* **187**, 339–343. Free vibration of a rectangular plate loaded by a non-uniform in-plane force.
3. S. Timoshenko and J. N. Goodier 1951 *Theory of Elasticity*. New York: McGraw-Hill.